Simpson's rule application for experimental determining parameters of rock mass oscillation equation at limestone

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SIMPSON'S RULE APPLICATION FOR EXPERIMENTAL DETERMINING THE PARAMETERS OF ROCK MASS OSCILLATION EQUATION AT LIMESTONE

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Abstract: *By its form, the use of explosives has a destructive character. However, energy of chemical explosives is tamed and perfected in many effects, so that is developed a series of activities which work is unthinkable without the use of explosives. In mining industry, explosives are most commonly used for exploitation in hard rock mass. Thereby, large amounts of explosives can be used, especially for mass blasting. The application of large amounts of explosives gives auspicious techno – economic indicators but also leads to increase the negative effects of blasting. By negative effects of blasting we understand the effect of the air blast wave, sound effect seismic, scattering of blasted rock effect of blasting and above all seismic effect of blasting which effects are monitored in this paper. To estimate, seismic effects of blasting it is necessary to determine rock mass oscillation equation. In this paper, analysis of the methods for the determination the parameters of the rock mass oscillation equation have performed, for the equal reduced distances at experimental blasting. To determine parameters, besides the usual model – Method of Least Squares, one more model has been presented applying Simpson's rule.*

Keywords: *working environment, seismic effects of blasting, oscillation velocity, rock mass oscillation equation, Simpson's rule*

1. INTRODUCTION

Shock wave intensity, which is caused by blasting operation, can be established by one of three basic dynamic parameters that characterize oscillation of aroused rock mass. Those are:

- displacement – distance at which the particle moves away from the equilibrium position during oscillation
- velocity – particle motion velocity during oscillation
- acceleration – shows the degree of change of the oscillation velocity, i.e. particles motion

Shock wave intensity, in the elastic deformations zone of the rock mass, is the most defined by the oscillation velocity. Different empirical equations, that define relation between rock mass oscillation velocity and amount of explosive, distance from the blasting point to the point of observation and way of blasting operation, is given in the literature.

As the relation between rock mass oscillation velocity and basic parameters affecting its magnitude, such as: the amount of explosive, the distance from the blast site, characteristics of the rock material and the type of blasting, the equation of M.A. Sadovskii is most commonly used.

By applying the equation of rock mass oscillation while blasting, the determination of the rock mass oscillation velocity is enabled for each blast operation in advance, which means that the magnitude of shock waves can be planned for each future blast operation. On that way, the negative blasting effects are reduced [1].

2. ROCK MASS OSCILLATION EQUATION

The equation that we most commonly use, in practice, is equation of M.A. Sadovskii, which is given in the following form [2]:

$$v = K \cdot R^{-n}; \tag{1}$$

where there are:

v – velocity of rock mass oscillation [cm/s],

K – coefficient conditioned by rock mass characteristics and blasting conditions, where the amount of explosive is given by way of the volume. K is determined by terrain surveying,

n – exponent, conditioned by rock mass properties and mining conditions and also determined by terrain surveying,

R – reduced distance, expressed as $R = \frac{r}{\sqrt[3]{Q}}$, where:

r – distance from the blast site to the monitoring point [m],

Q – amount of explosive [kg].

3. MODELS OF DETERMINING THE PARAMETERS OF EQUATION OF ROCK MASS OSCILLATION VELOCITY

In the equation (1) two parameters appear, K and n, which need to be determined for a specific work environment and specific blasting conditions.

3.1. Model 1 - determining the parameters by applying the Least Square Method

The Least Square Method is mainly used to obtain the parameters K and n, which represents the most commonly model [3].

3.2. Model 2 – determining parameters by applying Simpson's rule

In this paper, as model 2, we will use Simpson's formula to approximate the values of the definite interval functions [3-4].

Let the values for the negative function $y = y(x)$ be known as $y = y_0, y_1, y_2, \dots, y_{2m}$ for $x = x_0 + ih, i = 0,1,2, \dots, 2m; h = \frac{x_{2m} - x_0}{2m}$.

Then

$$\int_{x_0}^{x_{2m}} y(x)dx \approx \frac{h}{3}[y_0 + y_{2m} + 4(y_1 + y_3 + \cdots y_{2m-1}) + 2(y_2 + y_4 + \cdots + y_{2m-2})]; \tag{2}$$

Formula (2) is Simpson's formula for the finding approximate values of the definite integral function $y = y(x)$ in the given interval $[x_0,\ x_{2m}]$.

In order to verify this model, experimental blasting is performed. Ten explosions are registered during the blasting operation, with corresponding reduced distances and oscillation velocities.

For $R = R_i$ we have $v = v_i; i = 0, 1, 2, \dots, 9$; which means that according to Equation (1) we have

$$v_i = K \cdot R_i^{-n}, v_i = K \cdot R_i^{-n}; \tag{3}$$

Equation (3) is written in the following form:

$$R_i^n = \frac{K}{v_i}; \tag{4}$$

From where we have:

$$\left(\frac{R_0 \cdot R_1 \cdot \ldots \cdot R_4}{R_5 \cdot R_6 \cdot \ldots \cdot R_9}\right)^n = \frac{v_5 \cdot v_6 \cdot \ldots \cdot v_9}{v_0 \cdot v_1 \cdot \ldots \cdot v_4}; \tag{5}$$

By logarithm operation of Equation (5) we get

$$n log \left(\frac{R_0 \cdot R_1 \cdot \ldots \cdot R_4}{R_5 \cdot R_6 \cdot \ldots \cdot R_9}\right)^n = log \left(\frac{v_5 \cdot v_6 \cdot \ldots \cdot v_9}{v_0 \cdot v_1 \cdot \ldots \cdot v_4}\right); \tag{6}$$

From where we find:

$$n = \frac{log \left(\frac{v_5 \cdot v_6 \cdot \ldots \cdot v_9}{v_0 \cdot v_1 \cdot \ldots \cdot v_4}\right)}{log \left(\frac{R_0 \cdot R_1 \cdot \ldots \cdot R_4}{R_5 \cdot R_6 \cdot \ldots \cdot R_9}\right)}; \tag{7}$$

According to Simpson's formula (2) we have:

$$\int_{R_n}^{R_9} v dR \approx \int_{R_n}^{R_9} K R^{-n} dR \approx S_s; \tag{8}$$

Where S_s represents the right side in formula (2).

$$S_s = \frac{h}{3}\left[y_0 + y_{2m} + 4(y_1 + y_3 + \cdots y_{2m-1}) + 2(y_2 + y_4 + \cdots + y_{2m-2})\right]; \tag{9}$$

How it is:

$$\int_{R_0}^{R_9} K R^{-n} dR = \frac{K}{n-1}\left(\frac{1}{R_0^{n-1}} - \frac{1}{R_9^{n-1}}\right) = \frac{K}{n-1}\left[\frac{R_9^{n-1} - R_0^{n-1}}{(R_0 R_9)^{n-1}}\right] \approx S_s; \tag{10}$$

From Equation (10) we find:

$$K = \frac{(n-1) \cdot (R_0 R_9)^{n-1}}{R_9^{n-1} - R_0^{n-1}} \cdot S_s; \tag{11}$$

By substituting K from Equation (11) into Equation (1) we get the following formula:

$$v = \frac{(n-1)(R_0 R_9)^{n-1}}{R_9^{n-1} - R_0^{n-1}} \cdot S_s \cdot R^{-n}, n \neq 1; \tag{12}$$

For $n = 1$ Equation (1) is reduced to:

$$v = \frac{S_s}{log R_9 - log R_n} \cdot R^{-1}, n = 1; \tag{13}$$

4. DEFINING STATISTICAL CRITERIA

For the above-mentioned models 1 and 2, based on experimental data, we have obtained equations, which make possible to determine the oscillation velocities of the rock mass v depending on the reduced distance R.

In order to assess the degree of connection between $logR$ and $logv$, we have used the linear correlation coefficient r [5].

In order to assess the degree of connection between the reduced distance R and rock mass oscillation velocity v, we have used the curved line dependency index ρ [6].

5. METHODOLOGY OF RESEARCH

5.1. General characteristics of the work environment in Kijevo open pit

Experimental researches were performed on limestone at Kijevo open pit. Kijevo open pit is located in the Belgrade municipality of Rakovica, in Serbia [7]. By examining the physical and mechanical properties of the work environment, the following values were obtained:

- compressive strength $\sigma_p = 96\ [MPa]$;
- bending strength $\sigma_s = 18\ [MPa]$;
- tensile strength $\sigma_i = 9\ [MPa]$;
- cohesion $C = 17\ [MPa]$;
- strength coefficient $f = 10$;
- volume density $\gamma = 26\ [kN/m^3]$;
- angle of internal friction $\varphi = 54\ [°]$;
- P – wave velocity, $c_p = 4.0 \left[{}^{km}/_{h}\right]$
- S – wave velocity, $c_s = 1.0 \left[{}^{km}/_{h}\right]$

5.2. Method of blasting in Kijevo open pit

The measurements of shocks resulting from blasting in Kijevo open pit were performed during blasting using half-second electric detonators. Delay time of initiation between boreholes was 0.5 s, which led to ten explosions and appropriate rock mass oscillation velocities.

As explosive was used powdered ammonium nitrate for general purposes Amoneks-I manufactured by Trayal from Kruševac. The holes were arranged in a single line while one cartridge of explosive was placed in each hole. Diameter of explosive cartridge was 28 mm, cartridge length of 0.15 m, and cartridge weight of 0.1 kg. Electric capacitor was used for initiation of the explosive.

During experimental investigations performed in Kijevo open pit, the following blasting parameters were specified:

- depth of borehole: 0.5 [m];
- weight of explosive charge per borehole: 0.10 [kg];
- number of boreholes: 10;
- distance between boreholes: 1.0 [m];
- distance between measuring point and the first borehole: 15.0 [m];
- delay time between initiation of boreholes: 0.5 [s].

Seismographs are instruments used for seismic parameters observation. From seismograph, maximum values for three oscillation velocity components can be recorded in directions X, Y and Z from the rectangular coordination system, as follows:

- v_t – rock mass oscillation velocity horizontal transversal component,
- v_v – rock mass oscillation velocity vertical component,
- v_l – rock mass oscillation velocity horizontal longitudinal component.

Based on that, maximum, i.e. resulting rock mass oscillation velocity v_r can be determined using following equation:

$$v_r = \sqrt{v_t^2 + v_v^2 + v_l^2}; \tag{14}$$

6. CALCULATION FOR PARAMETERS OF ROCK MASS OSCILLATION EQUATION IN THE KIJEVO OPEN PIT

The values of distance from the blasting point to the point of observation r, amount of explosive Q, calculated values of reduced distances R, registered values of rock mass oscillation velocities by components v_t, v_v, v_l and resulting rock mass oscillation velocities v_r for a total of ten explosions, are given in Table 1.

Table 1. *Review of blasting parameters and measurement results*

No	r [m]	Q [kg]	R	$v_t[^{cm}/_{s}]$	$v_v[^{cm}/_{s}]$	$v_l[^{cm}/_{s}]$	$v_r[^{cm}/_{s}]$
1	15.0	0.1	32.3165	0.170	0.095	0.270	0.3329
2	16.0	0.1	34.4710	0.155	0.080	0.260	0.3131
3	17.0	0.1	36.6254	0.115	0.085	0.245	0.2837
4	18.0	0.1	38.7798	0.080	0.095	0.260	0.2881
5	19.0	0.1	40.9343	0.180	0.120	0.120	0.2474
6	20.0	0.1	43.0887	0.155	0.090	0.145	0.2305
7	21.0	0.1	45.2431	0.170	0.090	0.115	0.2241
8	22.0	0.1	47.3976	0.120	0.090	0.130	0.1985
9	23.0	0.1	49.5520	0.125	0.100	0.125	0.2031
10	24.0	0.1	51.7064	0.150	0.125	0.100	0.2194

Based on data given in Table 1, the rock mass oscillation equation is calculated by the formula (1) – by the models 1 and 2. The calculation of the curve was carried out for values of reduced distances from $R = 32.3165$ to $R = 51.7064$. Thus, curve parameters were calculated enabling us to determine the equation of rock mass oscillation in the form of:

- **Model 1**

$$v_1 = 15.4260 \cdot R^{-1.1060}; \tag{15}$$

whereby linear dependence between $log\ v$ and $log\ R$ has been obtained, expressed by the equation (15), with the linear correlation coefficient r amounting: $r = -0.9536$.
Graphic review of the rock mass oscillation equation is shown in figure 1.

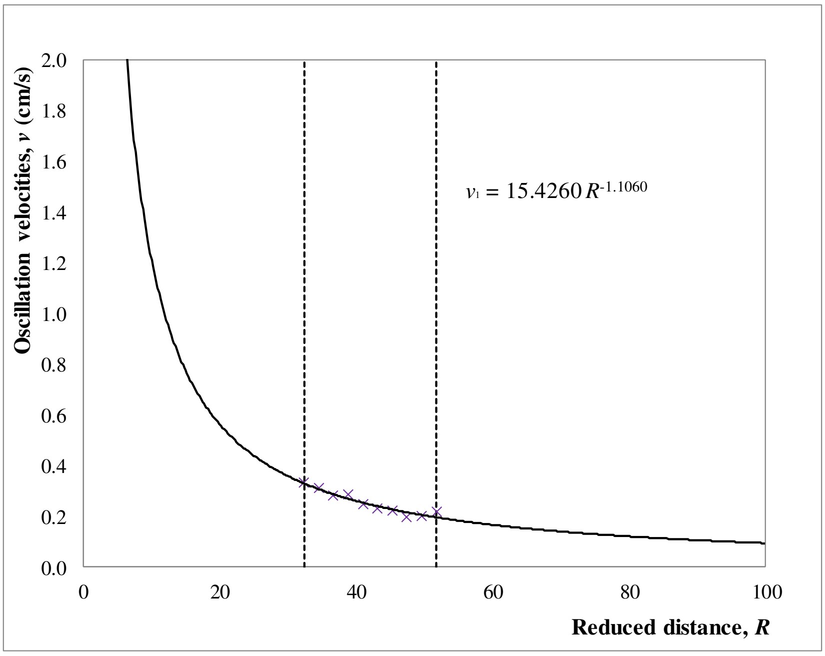

Figure 1. *Graphic review of oscillation velocity and reduced distance in the Kijevo open pit*

- **Model 2**

$$v_2 = 22.2039 \cdot R^{-1.1795}; \tag{16}$$

Based on the obtained equations for rock mass oscillation (15) and (16), it is possible to calculate values of rock mass oscillation velocities for the corresponding reduced distances for models 1 and 2. Review of reduced distances R, registered rock mass oscillation velocities v_r, calculated rock mass oscillation velocities v_{i1}, v_{i2}, as well as the difference between registered and calculated oscillation velocities for models 1 and 2 are given in Table 2.

Table 2. *Review of recorded and calculated rock mass oscillation velocities for models 1 and 2*

No	R	$v_r[{}^{cm}/_s]$	$v_{i1}[{}^{cm}/_s]$	$v_{i2}[{}^{cm}/_s]$	$v_r - v_{i1}$	$v_r - v_{i2}$
1	32.3165	0.3329	0.3305	0.3682	0.0024	-0.0353
2	34.4710	0.3131	0.3078	0.3412	0.0053	-0.0281
3	36.6254	0.2837	0.2878	0.3176	-0.0041	-0.0339
4	38.7798	0.2881	0.2702	0.2969	0.0179	-0.0088
5	40.9343	0.2474	0.2545	0.2786	-0.0071	-0.0312
6	43.0887	0.2305	0.2405	0.2622	-0.0100	-0.0317
7	45.2431	0.2241	0.2278	0.2476	-0.0037	-0.0235
8	47.3976	0.1985	0.2164	0.2343	-0.0179	-0.0359
9	49.5520	0.2031	0.2060	0.2224	-0.0029	-0.0193
10	51.7064	0.2194	0.1966	0.2115	0.0228	0.0079

Based on the data in Table 2, a statistical analysis was carried out and the following values were obtained:

- for model 1:

The curved line dependency index ρ_1 between the reduced distance R and rock mass oscillation velocity v is: $\rho_1 = 0.9650$ (there is a strong correlation between R and v, given in the formula (15)).

- for model 2:

The curved line dependency index ρ_2 between the reduced distance R and rock mass oscillation velocity v is: $\rho_2 = 0.7915$ (there is a strong correlation between R and v, given in the formula (16)).

7. CONCLUSION

As a relationship between rock mass oscillation velocity and basic parameters that influence on its value the equation of M.A. Sadovskii is the most used.

In this paper, parameters K and n in the Sadovskii equation have been determined by two ways – two models, in the given working environment. First model represents the usual method of the Least Squares, and the second model has been derived by applying Simpson's rule. Thus, we obtained the adequate functions which represent rock mass oscillation velocity depending on reduced distance. Based on calculated reduced distances, registered rock mass oscillation velocities, calculated rock mass oscillated velocities as well as differences between registered and calculated velocities, statistical analysis was performed.

Comparing values of the recorded oscillation velocities of the rock mass with the corresponding calculated ones, it is visible that they are approximately the same. Calculated corresponding indices of curvilinear correlation show that between the reduced distance and the rock mass oscillation velocity there is a significant and very strong curvilinear relationship expressed by the functions obtained.

Based on the values obtained by statistical analysis we conclude that this developed model can be used for equal reduces distances with application of Simpson's rule, at experimental blastings.

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